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Corrections to quark asymmetries at LEP

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Abstract. The most precise measurement of the weak mixing angle $\sin^2\theta^l_{\rm eff}$ at LEP is from the forward–backward asymmetry $e^+e^- \to b\bar{b}$ at the Z-pole. In this note the QED and electroweak radiative corrections to obtain the pole asymmetry from the measured asymmetry for b- and c-quarks have been calculated using ZFITTER, which has been amended to allow a consistent treatment of partial two-loop corrections for the b-quark final asymmetries. A total correction of $\delta A^b_{\rm FB} = 0.0019 \pm 0.0002$ and $\delta A^c_{\rm FB} = 0.0064 \pm 0.0001$ has been found, where the remaining theoretical uncertainty is much too small to explain the apparent discrepancy between $\sin^2\theta^l_{\rm eff}$ obtained from $A^b_{\rm FB}$ and from the left–right asymmetry at SLD.

1 Introduction

At LEP and SLD the effective electroweak mixing angle at the Z-scale, $\sin^2\theta^l_{\rm eff}$, can be measured using several different asymmetries [1]. The two most precise measurements of this quantity are obtained from the left–right asymmetry with a polarised electron beam at SLD and the forward–backward asymmetry for b-quarks at LEP. Both measurements provide a relative precision on $\sin^2\theta^l_{\rm eff}$ of around 10^{-3}. This is significantly more precise than the expected loop effects which allows for example the estimation of the Higgs boson mass from the electroweak precision data. On the other hand this very high precision also requires a good understanding of all higher order corrections like photon radiation, photon exchange, mass effects etc.

In this note the procedure to correct the measured forward–backward asymmetries for b-quarks will be described with special attention to the recent modifications.

2 Correction procedure

In the electroweak fits the experimental measurements are not used directly but instead so called pseudo-observables are used that are obtained from the measurements with some almost model independent corrections. For the b-asymmetry this pseudo-observable is the pole asymmetry $A^{0,b}_{\rm FB}$. This pole asymmetry can be viewed as the b-asymmetry on the Z-peak without photon exchange, QED and QCD corrections and taking only the real parts of the Z–fermion couplings. The weak mixing angle is then given in terms of the pole asymmetry as

$$A^{0,b}_{\rm FB} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_b, \tag{1}$$

$$\mathcal{A}_f = \frac{2g_{Vf}g_{Af}}{g^2_{Vf}+g^2_{Af}},$$

$$\frac{g_{Vf}}{g_{Af}} = 1 - 4Q_f\sin^2\theta^f_{\rm eff},$$

where g_V and g_A are the effective coupling constants of the weak neutral current.

The QCD corrections [2,3] arise mainly from the smearing of the event axis due to gluon radiation. Their size thus depends strongly on the experimental selection procedure. For this reason the QCD corrections are already performed by the experiments and corrected values are provided for combination. The procedure is described in detail in [2].

The energy dependence of the asymmetries is given by the contribution from the γ–Z interference. It is numerically large, but can be predicted with negligible theoretical uncertainty. In the combination procedure all experimental asymmetries are first corrected to a centre of mass energy of $\sqrt{s} = 91.26$ GeV assuming a standard model energy dependence as predicted by ZFITTER [4]. In a last step the LEP-combined value of $A^b_{\rm FB}(91.26\,{\rm GeV})$ is then corrected to the pole asymmetry $A^{0,b}_{\rm FB}$ [5]. For c-quarks exactly the same procedure is followed.

This correction is again done using ZFITTER. This program allows to calculate realistic observables and as well as pseudo-observables. The total correction is calculated as

$$\delta A^b_{\rm FB} = \left(A^{0,b}_{\rm FB} - A^b_{\rm FB}\right)_{\rm ZFITTER}$$

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so that the measured value of $A_{\rm FB}^{0,\,b}$ can be expressed as

$$A_{\rm FB}^{0,\,b}(\text{meas}) = A_{\rm FB}^{b}(\text{meas}) + \delta A_{\rm FB}^{b}.$$

For clarity the correction is split into three parts:
(1) an energy shift from $\sqrt{s} = 91.26\,\text{GeV}$ to $\sqrt{s} = m_Z$;
(2) QED corrections;
(3) other corrections including γ-exchange and γ–Z interference, mass effects and imaginary parts of couplings.

The QED corrections affect the asymmetries mainly by the change in centre of mass energy due to the initial state radiation. Since the average running energy of LEP was slightly above the Z-mass the QED corrections and the energy correction are partially cancelling. For these corrections it is also clear that they should be treated as an additive correction. With the small experimental error on the b asymmetry it makes, however, numerically no difference if the correction is treated as additive or multiplicative.

3 ZFITTER modifications

Recently, complete electroweak two-loop results for the prediction of the W-boson mass, M_W [6, 7], and exact fermionic results for the two-loop corrections to the effective leptonic weak mixing, $\sin^2\theta^l_{\rm eff}$, [8] became available. Here the fermionic two-loop corrections denote all two-loop contributions with at least one closed fermion loop. These results improve on the prediction for the precision pseudo-observables in the standard model with respect to the previously known partial results for electroweak two-loop corrections using an expansion for large values of the top-quark mass up to next-to-leading order [9]. These latter results had been incorporated into ZFITTER from version 5.10 upwards [4]. Complete two-loop results for the Z-boson partial widths are still missing.

The fermionic two-loop corrections to M_W [6] were implemented into ZFITTER in the version 6.36. The version 6.40 incorporates the complete two-loop corrections to M_W [7] including new partial three-loop corrections of order $\mathcal{O}(\alpha^3)$ and $\mathcal{O}(\alpha^2\alpha_{\rm s})$ [10]. In the same version also the fermionic two-loop corrections to the pseudo-observable $\sin^2\theta^l_{\rm eff}$ [8] are implemented. Internally, the pseudo-observables are computed in the subroutine ZWRATE in the package DIZET.

The interfaces ZUTHSM, ZUTPSM, ZULRSM and ZUATSM, on the other hand, calculate the cross-sections and asymmetries directly from the standard model predictions for the weak vertex form factor computed in the subroutine ROKANC [4]. It is important to observe that the weak form factors, denoted $\rho_{ef}(s,t)$, $\kappa_e(s,t)$, $\kappa_f(s,t)$ and $\kappa_{ef}(s,t)$, depend on the final-state fermion type f. The weak corrections for the $b\bar{b}$ final state are substantially different than for the other flavours, since the vertex loop corrections for the $Zb\bar{b}$ vertex involve heavy internal top-quark propagators. This peculiarity is consistently treated in ZFITTER up to one-loop order. Up to now, however, no two-loop results for the electroweak corrections to the $Zb\bar{b}$ vertex are available. This is already true for the previously known leading-m_t corrections [9]. In this case, i.e. for INDF $= 9$, all versions of ZFITTER up to 6.40 calculate *all* four form factors in ROKANC in one-loop approximation.

While this is the best possible treatment for the $Zb\bar{b}$ vertex that we can achieve today, it produces inconsistencies for the initial Ze^+e^- form factors when including electroweak two-loop corrections, i.e. for AMT4 ≥ 4. The reason is that in ROKANC the Ze^+e^- from factors for all other final states will be generated including two-loop corrections, while for the $b\bar{b}$ final state only one-loop corrections are used.

This mismatch also affects the ZFITTER interfaces ZUXSA, ZUTAU and ZUXSA2, which use the language of effective couplings [4], since they are defined to coincide exactly with the complete standard model prediction in ROKANC if the effective couplings coincide with their standard model analogue.

The problem has been alleviated in the newest version ZFITTER 6.41 [12]. In contrast the older implementations, $\kappa_e(s,t)$ and $\kappa_f(s,t)$ are not treated symmetrically anymore for INDF $= 9$, but two-loop electroweak corrections are included in $\kappa_e(s,t)$ for AMT4 ≥ 4, yet not in $\kappa_b(s,t)$. The treatment of $\rho_{ef}(s,t)$ and $\kappa_{ef}(s,t)$ has been changed accordingly. Here one can use the fact that the presently known two-loop contributions factorise into initial-state and final-state corrections. The changes in the code for ROKANC affect both the treatment of the previously available leading-m_t corrections for AMT4 $= 4$, as well as the new corrections for $\sin^2\theta^l_{\rm eff}$. Numerically these modifications lead to an upward shift of about 0.0006 for the prediction of $A^b_{\rm FB}$ compared to previous ZFITTER versions.

The new two-loop corrections [6–8], which do not rely on a large-mass expansion, can be accessed through the flag AMT4. The setting AMT4 $= 5$ corresponds to the status of the version ZFITTER 6.36, which includes the complete fermionic corrections to the W-mass [6] and has been used for the summer 2001 LEP electroweak fits [11]. The assignment AMT4 $= 6$, used from summer 2004 onwards, enables the calculation of M_W including complete two-loop and leading three-loop corrections [7, 10] and the inclusion of the new fermionic two-loop corrections to the effective weak mixing angle. The estimated theoretical uncertainties for these two quantities can be simulated by varying the flags DMWW (for M_W and AMT4 $= 5, 6$) and DSWW (for $\sin^2\theta^l_{\rm eff}$ and only AMT4 $= 6$) between -1 and 1, respectively.

4 Results

The corrections to the quark asymmetries are summarised in Table 1. For comparison also the corrections for s-quarks are given. For these values the full two-loop corrections on $\sin^2\theta^l_{\rm eff}$ (AMT4 $= 6$) are used. In this case it is assumed that the couplings of the $Zf\bar{f}$ vertex factorise between the initial and the final state. The flavour specific corrections for the $Zb\bar{b}$ vertex are not yet calculated, however they are highly suppressed because of the small b-quark charge and because $\mathcal{A}_b$ is so close to 1 (see (1)). For c- and s-quarks

Table 1. Corrections to be applied to the quark asymmetries as $A_{\mathrm{FB}}^{0,q} = A_{\mathrm{FB}}^{q}(\mathrm{pk}) + \delta A_{\mathrm{FB}}$. "other" denotes corrections due to γ-exchange, γ–Z interference, quark-mass effects and imaginary parts of the couplings

Source	$\delta A_{\mathrm{FB}}^{c}$	$\delta A_{\mathrm{FB}}^{b}$	$\delta A_{\mathrm{FB}}^{s}$
$\sqrt{s} = m_Z$	−0.0035	−0.0014	−0.0014
QED corrections	+0.0107	+0.0039	+0.0038
other	−0.0008	−0.0006	−0.0003
Total	+0.0064	+0.0019	+0.0021

no approximations are involved. Up to now corrections of $\delta A_{\mathrm{FB}}^{b} = 0.0025$ and $\delta A_{\mathrm{FB}}^{c} = 0.0062$ have been used [1]. While there is no significant change for A_{FB}^{c}, for the b-asymmetry the total correction is 0.0006 below that value so that the LEP-combined result of $A_{\mathrm{FB}}^{0,b}$ will decrease by that amount.

To verify that these corrections are reliable several cross checks have been made. If the Higgs mass is varied between 100 GeV and 1 TeV the corrections stay constant. The same is true if the top mass and $\alpha(m_Z)$ are varied within several standard deviations. Also when instead of the full two-loop corrections only the leading corrections by Degrassi et al. [9] are used, that are implemented in ZFITTER since long (AMT4 = 4) none of the values in Table 1 change.

If one uses only the full one-loop corrections to $\sin^2\theta_{\mathrm{eff}}^{l}$ (AMT4 = 3) the b-quark treatment in ZFITTER is exact. In this case the total correction increases by 0.00015. However the difference between the b-quark and the s-quark correction stays constant showing that this is a genuine two-loop effect and not an artefact of the involved approximations.

With the public version of TOPAZ0 [13] it is not possible to reproduce Table 1, since final-state QCD corrections cannot be switched off. However the initial- and final-state deconvoluted asymmetries as well as $A_{\mathrm{FB}}^{0,b}$ using TOPAZ0 are given in [14] (Tables 6 and 14). From these values the critical correction labelled "other" in Table 1 can be calculated to be −0.0005, well in agreement with the ZFITTER value.

4.1 Systematic uncertainties

To assess the systematic uncertainty from QED corrections the relevant flags in ZFITTER have been varied. The only flag for which a variation of the result has been found was FBHO, which describes the treatment of fermion pair radiation in the asymmetry calculation. From this an error of

$$\Delta(\delta A_{\mathrm{FB}}^{b})(\mathrm{QED}) = 0.00017,$$
$$\Delta(\delta A_{\mathrm{FB}}^{c})(\mathrm{QED}) = 0.00011,$$

has been derived. Especially no uncertainty from the choice of the radiator function (flag FOT2) has been found.

For the b-quark the flavour specific two-loop corrections have not been calculated yet. As an estimate of the uncertainty from these diagrams the difference between the total correction for b- and s-quarks has been used leading to

$$\Delta(\delta A_{\mathrm{FB}}^{b})(2 - \mathrm{loop}\, b) = 0.00016\,.$$

To test the uncertainty due to the universal higher order corrections the flags DMWW and DSWW mentioned in Sect. 3 have been varied and no significant change in the asymmetry corrections has been seen. This results in a total uncertainty of the QED and electroweak corrections to the b- and c-quark asymmetry of

$$\Delta(\delta A_{\mathrm{FB}}^{b}) = 0.0002,$$
$$\Delta(\delta A_{\mathrm{FB}}^{c}) = 0.0001.$$

5 Conclusions

After correcting some inconsistencies in the treatment of b-quarks in ZFITTER, the QED and electroweak corrections to obtain the pole asymmetry from the measured, QCD corrected, forward–backward asymmetry at the Z-peak have been calculated. Total corrections of

$$\delta A_{\mathrm{FB}}^{b} = 0.0019 \pm 0.0002,$$
$$\delta A_{\mathrm{FB}}^{c} = 0.0064 \pm 0.0001,$$

have been found. The total corrections are only slightly larger than the experimental errors while their uncertainties are about an order of magnitude smaller. It is thus inconceivable that these corrections can explain the apparent difference in $\sin^2\theta_{\mathrm{eff}}^{l}$ obtained from the left–right asymmetry at SLD and from A_{FB}^{b}.

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References

1. The LEP collaborations, A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model, CERN-EP/2003-091, hep-ex/0312023
2. LEP Heavy Flavour Working Group Collaboration, D. Abbaneo et al., Eur. Phys. J. C **4**, 185 (1998)
3. The LEP Heavy Flavour Group, Final input parameters for the LEP/SLD heavy flavour analyses, LEPHF/01-01, http://www.cern.ch/LEPEWWG/heavy/lephf0101.ps.gz
4. D.Y. Bardin et al., Comput. Phys. Com. **133**, 229 (2001)
5. ALEPH, DELPHI, L3, OPAL Collaboration, Nucl. Instrum. Methods A **378**, 101 (1996)
6. A. Freitas, W. Hollik, W. Walter, G. Weiglein, Phys. Lett. B **495**, 338 (2000) [Erratum B **570**, 260 (2003)]; Nucl. Phys. B **632**, 189 (2002) [Erratum B **666**, 305 (2003)]; M. Awramik, M. Czakon, Phys. Lett. B **568**, 48 (2003)
7. M. Awramik, M. Czakon, Phys. Rev. Lett. **89**, 241801 (2002); A. Onishchenko, O. Veretin, Phys. Lett. B **551**, 111 (2003); M. Awramik, M. Czakon, A. Onishchenko, O. Veretin, Phys. Rev. D **68**, 053004 (2003); M. Awramik, M. Czakon, A. Freitas, G. Weiglein, Phys. Rev. D **69**, 053006 (2004)

8. M. Awramik, M. Czakon, A. Freitas, G. Weiglein, Phys. Rev. Lett. **93**, 201805 (2004) [hep-ph/0407317]; see also Nucl. Phys. Proc. Suppl. **135**, 119 (2004) [hep-ph/0408207]; hep-ph/0409142
9. G. Degrassi, P. Gambino, A. Vicini, Phys. Lett. B **383**, 219 (1996); G. Degrassi, P. Gambino, A. Sirlin, Phys. Lett. B **394**, 188 (1997); G. Degrassi, P. Gambino, Nucl. Phys. B **567**, 3 (2000)
10. M. Faisst, J.H. Kühn, T. Seidensticker, O. Veretin, Nucl. Phys. B **665**, 649 (2003)
11. The LEP EWWG and SLD Heavy Flavour and Electroweak Groups, D. Abbaneo et al., hep-ex/0112021
12. /afs/cern.ch/user/b/bardindy/public/ZF6_41.
13. G. Montagna et al., Comput. Phys. Com. **117**, 278 (1999)
14. D.Y. Bardin, M. Grünewald, G. Passarino, hep-ph/9902452